Problem 2.1 ECEN Dr. Vashaee

Transmission Line: length l

sinusoidal voltage source frequency f

Velocity of propagation c

The line can be neglected if $\frac{l}{\lambda} < \frac{1}{100}$

or $\frac{\lambda}{l} > 100$, or $\lambda > 100\,l$

$\lambda = \frac{c}{f} \Rightarrow \frac{l}{\lambda} = \frac{lf}{c} = \frac{lf}{3\times10^8}$

(a) $l = 20$ cm, $f = 20$ kHz $\quad \frac{l}{\lambda} = \frac{(20\times10^{-2})(20\times10^{3})}{3\times10^{8}} = \frac{40}{3}\times10^{-6}$

$\because \frac{l}{\lambda} \ll 10^{-2}$ $\therefore$ The line can be neglected

(b) $l = 50$ km, $f = 60$ Hz $\quad \frac{l}{\lambda} = \frac{(5\times10^{4})\,60}{3\times10^{8}} = \frac{300\times10^{4}}{300\times10^{6}} = 10^{-2}$

This is just at the threshold, borderline

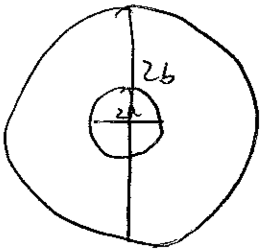
(c) $l = 20$ cm, $f = 600$ MHz $\quad \frac{l}{\lambda} = \frac{(20\times10^{-2})(6\times10^{8})}{3\times10^{8}} = 40\times10^{-2} = \frac{2}{5}$

$\frac{l}{\lambda}$ is 40 times higher than the threshold, so it can not be neglected

(d) $l = 1$ mm, $f = 100$ GHz $\quad \frac{l}{\lambda} = \frac{(10^{-3})(10^{11})}{3\times10^{8}} = \frac{1}{3}$

$\frac{l}{\lambda} = \frac{1}{3} > \frac{1}{100}$ $\therefore$ can not be neglected

Problem 2.2



Coaxial Line

Inner conductor diameter = 0.5 cm

Outer conductor diameter = 1 cm

Copper $\mu_c = \mu_0$ $\sigma_c = 5.8 \times 10^7$ S/m

Insulator : $\mu = \mu_0$, $\varepsilon_r = 4.5$ $\sigma = 10^{-3}$ S/m

Operating frequency = 1 GHz

$2a = \frac{1}{2} \times 10^{-2}$ $a = 0.25 \times 10^{-2}$

$2b = 1 \times 10^{-2}$ $b = 0.5 \times 10^{-2} = 2a$

ref table 2.1, P41 Eq 2.5 – 2.11 P42

$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$ $R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \left[\frac{\pi(10^9)(4\pi \times 10^{-7})}{5.8 \times 10^7}\right]$

$R' = \frac{8.25 \times 10^{-3}}{2\pi}(400 + 200)$ $R_s = (6.81 \times 10^{-5})^{1/2} = 8.25 \times 10^{-3}$

$R' = 0.7878$ ohms/meter

$L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{4\pi \times 10^{-7}}{2\pi}\ln(2) = 1.3863 \times 10^{-7}$ H/m

$G' = \frac{2\pi\sigma}{\ln\left(\frac{b}{a}\right)} = \frac{2\pi(10^{-3})}{\ln 2} = 9.0647 \times 10^{-3}$ mS/m

$C' = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)} = \frac{2\pi(4.5)(8.854 \times 10^{-12})}{\ln 2} = 361$ pF/m

Problem 2.3

$f = 1\,GHz$ width $= 1.2\,cm = w$

thickness $= 0.15$ cm $= d$

conductors (copper) $\mu_{rc} = 1$ $\sigma_c = 5.8 \times 10^7$ S/m

dielectric: $\varepsilon_{rd} = 2.6$ $\mu_{rd} = 1$ $\sigma_d = 0$ (polystyrene)

(Be careful with the table 2.1 formulae for the parallel plate line, the w is width not angular frequency, ω!)

$$R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \sqrt{\frac{(\pi \times 10^9)(4\pi \times 10^{-7})}{5.8 \times 10^7}} = \sqrt{68.0} \times 10^{-6} = 8.25 \times 10^{-3}$$

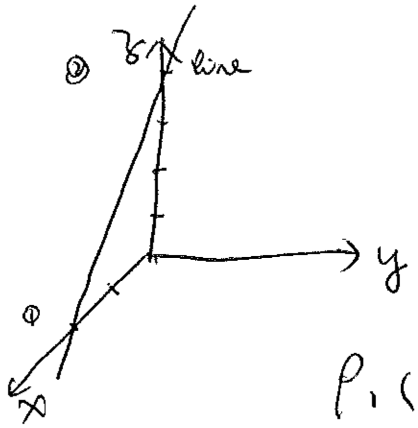
$$R' = \frac{2R_s}{w} = \frac{2}{0.012}(8.25 \times 10^{-3}) = 1.38\ \Omega/m$$

$$L' = \frac{\mu_{rd}\mu_0 d}{w} = \frac{(4\pi \times 10^{-7})(0.0015)}{0.012} = 1.571 \times 10^{-7} = 0.157\,\mu H/m$$

$$G' = \frac{\sigma_d w}{d} = 0 \quad S/m$$

$$C' = \frac{\varepsilon_{rd}\varepsilon_0 w}{d} = \frac{(2.6)(8.85 \times 10^{-12})(0.012)}{0.0015} = 184.1 \times 10^{-12} = 184.1\,pF/m$$

Problem 3.11



line: $2x + z = 4$

① at $z=0$, $x=2$

② at $x=0$, $z=4$

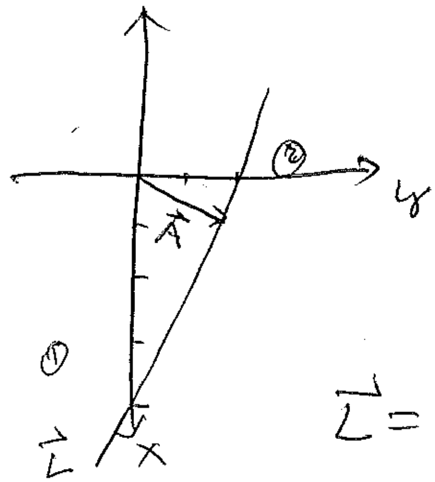
$P_1(2,0,0)$ $P_2(0,0,4)$

$$\vec{P}_{12} = \vec{P}_1 - \vec{P}_2 = 2\hat{a}_x - 4(\hat{a}_z) = 2\hat{a}_x - 4\hat{a}_z$$

$$\hat{a}_{P_{12}} = \frac{\vec{P}_{12}}{P_{12}} = \frac{2\hat{a}_x - 4\hat{a}_z}{\sqrt{4+16}} = \frac{(\hat{a}_x - 2\hat{a}_y)}{\sqrt{5}}$$

$$\hat{a} = \frac{\sqrt{5}}{5}(\hat{a}_x - 2\hat{a}_z) \text{ or } \frac{2\hat{a}_x - 4\hat{a}_z}{\sqrt{20}}$$

Problem 3.13



line: $x + 2y = 4$

① at $y=0$, $x=4$

② at $x=0$, $y=2$

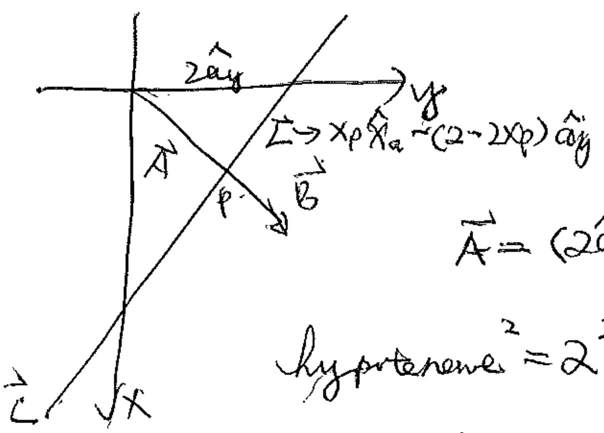
$P_1(4,0,0)$, $P_2(0,2,0)$

$\vec{L} = 4\hat{a}_x - 2\hat{a}_y \qquad L = \sqrt{20} \qquad \hat{a}_L = \dfrac{4\hat{a}_x - 2\hat{a}_y}{\sqrt{20}}$

desire $\vec{B}$ such ~~that~~ that $\vec{B}\cdot\vec{L} = 0$

$\Rightarrow 4B_x - 2B_y = 0 \Rightarrow 2B_x = B_y$

$\vec{B} = \hat{a}_x + 2\hat{a}_y$



$\vec{A} = x_p\hat{a}_x + y_p\hat{a}_y$

from above we know ~~that~~ $A_y = 2A_x$

$\Rightarrow \vec{A} = x_p\hat{a}_x + 2x_p\hat{a}_y$

$\vec{A} = (2\hat{a}_y) + x_p\hat{a}_x - 2(1-x_p)\hat{a}_y \; (x_p\hat{a}_x + 2x_p\hat{a}_y)$

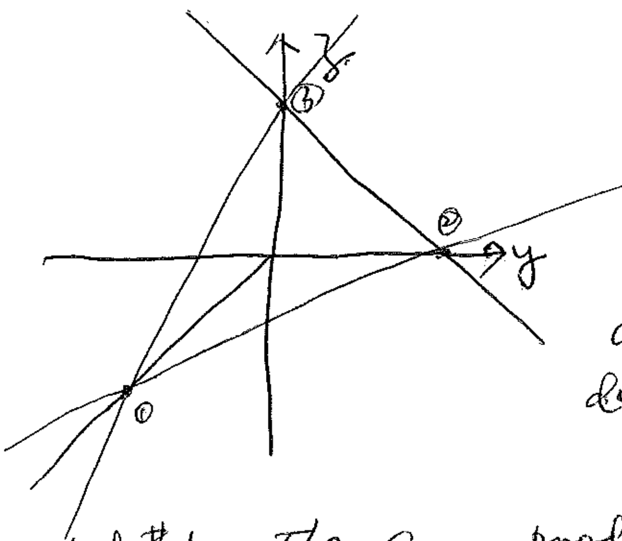
$\text{hypotenuse}^2 = 2^2 = \overbrace{(x_p^2 + 4x_p^2)}^{|A|^2} + \overbrace{[x_p^2 + 4(1-x_p)^2]}^{|C|^2}$

$4 = 6x_p^2 + 4 - 8x_p + 4x_p^2$, $10x_p^2 - 8x_p = 0$

$x_p(10x_p - 8) = 0 \qquad \therefore x_p = 0, \; x_p = \frac{4}{5}$

$\vec{A} = 0.8\hat{a}_x + 1.6\hat{a}_y$

Problem 3.15



Plane: $2x+3y+4z=16$

at ① $y, z=0$ $\quad x=8$, $P_1(8,0,0)$

② $x, z=0$ $\quad y=\frac{16}{3}$, $P_2(0,\frac{16}{3},0)$

③ $x, y=0$ $\quad z=4$, $P_3(0,0,4)$

④ $x=3, y=2, z=1$ $\quad P_4(3,2,1)$

any two lines in the plane define the plane

method #1 — The cross product of any two lines in the plane yield a vector normal to the plane (then get the unit vector in the $+x, +y, +z$ direction

$\vec{A} = \vec{P_{12}} = 8\hat{a}_x - \frac{16}{3}\hat{a}_y$ $\qquad \vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 8 & -\frac{16}{3} & 0 \\ 8 & 0 & -4 \end{vmatrix} = \frac{64}{3}\hat{a}_x + 32\hat{a}_y + \frac{128}{3}\hat{a}_z$

$\vec{B} = \vec{P_{13}} = 8\hat{a}_x - 4\hat{a}_z$

$\Rightarrow C = 57.44$ $\qquad \therefore \hat{a}_c = 0.37\hat{a}_x + 0.56\hat{a}_y + 0.74\hat{a}_z$

method #2 — Each line in the plane must be perpendicular to $\vec{C}$, so their dot products must both be zero; Then, using $\vec{A}+\vec{B}$ again, Assume $\vec{C} = C_x\hat{a}_x + C_y\hat{a}_y + C_z\hat{a}_z$

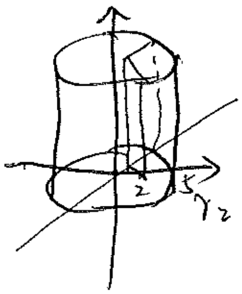
$\Rightarrow \vec{A}\cdot\vec{C} = \cancel{8x}\ 8C_x - \frac{16}{3}C_y = 0$, $\quad \vec{B}\cdot\vec{C} = 8C_x - 4C_z = 0$

$\left.\begin{array}{l} 3C_x = 2C_y \\ 2C_x = C_z \end{array}\right\} \Rightarrow \vec{C} = C_x\hat{a}_x + \frac{3}{2}C_x\hat{a}_y + 2C_x\hat{a}_z$ $\qquad C = C_x(1+\frac{9}{4}+4)^{\frac{1}{2}} = 2.69C_x$

$\hat{a}_c = \frac{\vec{C}}{C}$ So the C_x cancels,

$\hat{a}_c = 0.37\hat{a}_x + 0.56\hat{a}_y + 0.74\hat{a}_z$

Problem 3.23



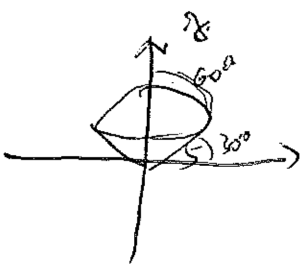
(a) $2 \le r \le 5$; $\frac{\pi}{2} \le \phi \le \pi$; $0 \le z \le 2$

$dv = r\,dr\,d\phi\,dz$

$V = \int_V dv = \int_0^2 \int_{\frac{\pi}{2}}^{\pi} \int_2^5 r\,dr\,d\phi\,dz$

$= 2 \left(\frac{\pi}{2}\right)\left(\frac{r^2}{2}\right)\Big|_2^5$

$\therefore V = \frac{\pi}{2}(25-4) \Rightarrow V = \frac{21\pi}{2}$



(b) $0 \le R \le 5$ $\quad 0 \le \theta \le \frac{\pi}{3}$ $\quad 0 \le \phi \le 2\pi$

$dv = R^2 \sin\theta\, dR\, d\theta\, d\phi$

$V = \int_V dv = \int_0^{2\pi} \int_0^{\pi/3} \int_0^5 R^2 \sin\theta\, dR\, d\theta\, d\phi$

$= \cancel{2\pi} \int_0^5 R^2 \cos\theta \Big|_{\pi/3}^{0} dR = \pi \frac{R^3}{3}\Big|_0^5$

$V = \frac{125\pi}{3}$

Problem 3.25

$$\vec{E} = r\cos\varphi\,\vec{a}_r + r\sin\varphi\,\hat{a}_\varphi + z^2\hat{a}_z$$

$P(2, \pi, 3)$ is a point on the cylindrical surface $r = 2$

(a) The surface vector at point $(2, \pi, 3)$ is $\tilde{a}_r$, so the components of $\vec{E} \perp$ to the cylindrical surface at point P can be found using the dot product:

$$\vec{E}\cdot\vec{a}_r\Big|_{(2,\pi,3)} = r(\cos\varphi)\Big|_{(2,\pi,3)} = 2\cos\pi = -2$$

so $\vec{E}_{np} = -2\hat{a}_r$

(b) Since $\vec{E}$ is expressed in the orthogonal cylindrical components, any part of $\vec{E}$ which is not normal to the surface, is tangential to it.

$\vec{E}_t = \vec{E} - \vec{E}_n \qquad \therefore \vec{E}_t = r\sin\varphi\,\hat{a}_\varphi + z^2\hat{a}_z$

and at point P, $\vec{E}_{tp} = 2\sin\pi\,\hat{a}_\varphi + (3)^2\,\hat{a}_z$

$\therefore \vec{E}_{tp} = 9\,\hat{a}_z$